AMERICAN BANKRUPTCY INSTITUTE
JOURNAL
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Use and Abuse of Quantitative Bankruptcy Prediction Models

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Rabbi Yochanan, a Jewish Talmudic scholar, was once quoted as saying, "From the day that the Holy Temple was destroyed, *prophecy* was given to fools." In non-religious realms, prophecies are made all of the time. In numerous fields and areas, so-called "experts" make projections, predictions and forecasts as if they can predict the future.

In sports, commentators, analysts and gamblers try to predict the result prior to the game and while the game is ongoing. For example, during Super Bowl LI, the New England Patriots were losing by 25 points with two minutes and 12 seconds remaining in the third quarter. Predictive models, commentators and spectators alike were giving the Patriots a near 0 percent chance to win the game — yet somehow, they did. Similarly, in politics, pollsters and other political "experts" said that there was a small probability that Donald Trump would win the 2016 presidential election, yet he did. We all tend to forget that Talmudic wisdom, and bankruptcy professionals are no exception.

In certain bankruptcy and nonbankruptcy litigation, parties argue, for example, that the lender, at the issuance of a loan, "knew or should have known" that the subject company had virtually no chance of being able to repay its debts and that it would be rendered insolvent if it borrowed the funds. This same logic is sometimes prevalent in certain tax court litigations.

One methodology to predict a company's ability to repay its debt, and ultimately its solvency, is to determine through careful, thorough analysis the company's ability to generate sufficient cash flows to service its debt obligations. In order to have a good understanding of a company's cash flows, financial analysts examine the key elements of its forecasts. This includes reviewing the company's historical performance, the key drivers for its business units, analyzing the industry and examining other key inputs in its cash flows. Some professionals elect to rely on more quantitative and academic/science-based methodologies that are supposedly more accurate due to their "scientific" nature.

However, these quantitative methodologies to estimate a company's probability of default are heavily dependent on certain assumptions. If the assumptions are unreasonable, the whole model becomes unreliable. Furthermore, the discretion given to the financial expert applying those methodologies can lead to the abuse of the model in both bankruptcy and nonbankruptcy litigation settings.

This article explains the basic idea behind bankruptcy prediction models and the various inputs that drive them. It begins with Altman's Z score, which utilizes a statistical analysis called the *multiple discriminant analysis*. Next, the article discusses the Merton model, a statistical model primarily used by academics and credit-rating agencies.[1] Finally, using data from a recent trial, the authors examine the vulnerabilities of the Merton model, emphasizing the importance of the bankruptcy professional to review all inputs carefully when relying on or confronted with this analysis.

Multiple Discriminant Analysis

One of the first quantitative-based methodologies used to determine the probability that a company will enter bankruptcy is the Altman Z score.[2] It is a bankruptcy-prediction model that utilizes traditional financial ratios and a statistical technique known as multiple discriminant analysis. The for-

1 This article only discusses the effectiveness and sensitivity of the model to its various inputs. Moody's Analytics has a bankruptcy prediction model referred to as the KMV-Merton model, which is based on the Merton model. However, in this article, the authors do not express an opinion on the commercial value of the KMV-Merton model as applied by rating agencies such as Moody's.

2 The Altman Z score is named after Prof. Edward Altman of New York University. He first published an article on the Z score in 1968, and the Z score equation has since been modified and updated by Prof. Altman and other academics.

mula for applying this methodology is $Z = 1.2(X_1) + 1.4(X_2) + 3.3(X_3) + 0.6(X_4) + 1.0(X_5)$, where X_1 equals working capital/total assets; X_2 equals retained earnings/total assets; X_3 equals earnings before interest and taxes (EBIT)/total assets; X_4 equals market value of the equity/book value of the total liabilities; and X_5 equals sales/total assets.

A multiple discriminant analysis is a statistical technique used to narrow the differences among numerous variables in order to categorize them into broad groups. In simplified terms, a discriminant analysis attempts to set a simple rule to classify data and provide the most meaningful separation.[3] The coefficients in the previous equation are the output of the statistical analysis and are used to assess whether a company is likely to go bankrupt.

Inputting the financial ratios for the variables in the equation above calculates a Z score, which ultimately indicates the company's likelihood of defaulting on its debts. According to the model, the lower the Z score, the more likely that a company is going to enter default. A Z score of less than or equal to 1.8 suggests a high probability of bankruptcy, while a Z score above or equal to 3.0 results in a low probability of bankruptcy. Any score in between 1.8 and 3.0 results in an "unsure" probability of default.[4]

The multiple discriminant analysis previously outlined has its strengths and weaknesses. Its primary strength is that it allows companies' credit scores to be compared across different industries. The model brings together different financial ratios that individually might not be comparable, but when brought together using the Z score, allow for a more comparable metric.

However, there are several weaknesses to this methodology. First, some of the financial ratios used in the calculation are fundamentally limited in their usefulness.

For example, "retained earnings" is an accounting measure and does not reflect a company's ability to generate profits and cash flow from an economic perspective. In addition, a company's sales or EBIT are not necessarily an indication of a company's ability to generate cash flow to service its debt obligations. A company can have large sales figures but may only generate profit margins of 1-2 percent. A company may also generate significant EBIT, but might require large capital expenditures and/or funding of its working capital, resulting in lower cash flow.

Therefore, a Z score might not necessarily be relevant to assess a company's ability to generate cash flow and repay its debt. Moreover, like any other model, the multiple discriminant analysis is only as good as its assumptions. Bankruptcy professionals should pay close attention to the reasonableness of the assumptions that drive the model and the conclusions that it ultimately reaches.

Finally, even if the inputs into the multiple discriminant analysis are reasonable, the output from the model is vague. As previously described, the calculated Z score is not precise and results in wide-ranging default categorizations such as "highly likely," "unlikely" or "unsure." Therefore, even though there is a comparability benefit of the multiple discriminant analysis, the imprecise nature of the inputs and the ambiguous output make it of limited usefulness.

3 Edward I. Altman, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," *Journal of Finance*, September 1968.

4 John Graham, Scott B. Smart and William L. Megginson, *Corporate Finance: Linking Theory to What Companies Do* (3d ed.), pp. 861-62 (South-Western Cengage Learning 2010).

The Merton Model

A more scientifically/quantitatively based bankruptcy prediction model is the Merton model, which has been used by numerous academics and credit-rating agencies. It is primarily based on Merton's bond-pricing model.[5]

The Merton model forecasts the subject company's probability of default at a given point in time. This is done by utilizing Merton's bond-pricing model, which establishes that the "equity of a firm is a call option on the underlying value of the firm with a strike price equal to the face value of the firm's debt."[6] Practically, as will be shown later, the Merton model is an equation with two unknown variables: the underlying value and volatility of the firm's assets. In order to solve for these two unknown variables, the model utilizes the firm's equity value and volatility, as well as other known variables.[7]

As with most financial applications, the Merton model is only as good as its assumptions. Since the model solves for two unknown variables, it becomes more sensitive to its inputs. Moreover, the model assumes that the underlying value of the subject company's assets follows a Geometric Brownian Motion. First applied in the world of physics, Brownian Motion is the zig-zagging motion exhibited by a small particle, such as a grain of pollen, immersed in a liquid or gas.[8] In other words, the model assumes that the value of the subject company's assets, though usually they have a positive drift, move at random. This assumption, along with the numerous other inputs required for applying the model, add to the unreliability of the methodology. The formula, along with its assumptions, will be discussed in more detail later in this article. The basic Merton model equation is shown in Exhibit 1.

The term *d*, as defined in Exhibit 1, is used along with a standard normal distribution to derive an estimated default rate for the subject company. Knowing that most bankruptcy professionals do not have a Ph.D. in physics, here is an explanation of the model in plain English. Exhibit 2 presents graphically how the Merton model generally functions. The solid red line represents the subject company's debt, growing at the debt's interest rate. The dashed blue line represents the subject company's asset value, growing at an assumed return on assets, fluctuating at an assumed asset volatility. Both values

5 Robert C. Merton, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *The Journal of Finance*, May 1974.

6 Sreedhar T. Bharath and Tyler Shumway, "Forecasting Default with the KMV-Merton Model," AFA 2006 Boston Meeting Paper, Dec. 7, 2004, p. 2.

7 *Id.*

8 Steven R. Dunbar, "Stochastic Processes and Advanced Mathematical Finance: The Definition of Brownian Motion and the Wiener Process," 2016, p. 2.

Exhibit 1: Basic Merton Model Equation

$$d = \frac{ln\left(\frac{v_t}{f_t}\right) + (\mu + 0.5\sigma_v^2)T}{\sigma_v\sqrt{T}}$$

V_t = Total asset value at time t
f_t = Default threshold at time t
μ = Expected return on assets
σ_v = Volatility of assets
T = Length of time period

are moving along the x-axis toward time *T*. If the asset value (the dotted blue line) crosses the value of the subject company's debt (the solid red line), it is considered to be in default.

Merton Model Equation

Asset Volatility

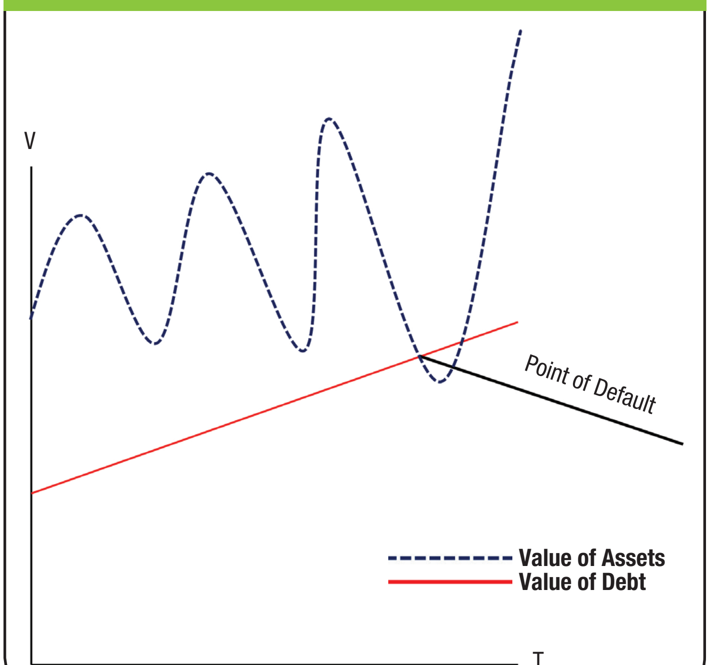
One of the key inputs of this model is the subject company's asset volatility, represented as σ_v in the equation shown in Exhibit 1. However, asset volatility is generally not directly observable for both privately held and publicly traded companies. If the subject company is publicly traded, one can analyze the volatility of the company's share price. By using simple mathematics, the company's equity value, standard deviation of the market return on equity and its debt balance, it is possible to estimate the company's asset volatility.

However, if the company is privately held, it is impossible to directly measure its equity volatility, because it does not trade in a public market. One can analyze publicly traded peer companies to determine a proxy for the subject company's equity volatility, but the calculation becomes far less reliable when this is undertaken. Hypothetically, if one were to use peer companies to estimate the subject company's asset volatility, pay close attention to the composition of the subject company's assets relative to the peers. For example, all else being equal, a company whose assets are comprised of 80 percent cash and 20 percent real estate will exhibit vastly different volatility than a company whose assets are 80 percent real estate and 20 percent cash. With that being said, relying on peer companies to determine the subject company's asset volatility adds a high degree of unreliability with respect to the model's results.

Return on Assets

Another essential assumption in the Merton model is the subject company's return on assets. In the previous equation, return on assets is represented by μ. Since the Merton model is a forward-looking analysis, the return on assets assumption should represent the subject company's expected return going forward from the bankruptcy assessment date. Ideally, there are forecasts available on the subject company's expected return on its assets. However, that is not always the case, and a financial analyst might have to look to the subject company's historical return on assets as an indication of future returns. Similar to asset volatility, information might not be available on the subject company's predicted or historical return on assets. An option is to analyze peer companies for an indication of the subject company's expected return on assets. However, this model becomes less reliable if this method is implemented.

Exhibit 2: General Function of the Merton Model



Time Period

An important input that is often overlooked in the Merton model is the time period (*T*) over which the probability of default is calculated. The longer the time period being utilized, the less reliable the results of the model will become. Probability of default models, such as the Merton model, should not be applied for a period longer than one year. For example, a public company might go no longer than one year without receiving an audit of its financials, and as a result, lenders would not go more than one year without the option to recalculate the probability of default of its borrowers. Moreover, by the model design, because the amplitude of the subject company's asset volatility increases as time *T* increases, implementing the Merton model over long time periods would render even the most creditworthy companies insolvent.

Case Study: The Courtroom Experience

All three of these inputs can potentially have a material impact on the conclusions drawn from the Merton model. A good example is the recent tax court decision in *Dynamo Holdings Limited Partnership, Dynamo GP Inc. v. Commissioner of Internal Revenue*.

In this case, the Internal Revenue Service challenged advances made between related parties, claiming that they should not be treated as debt but rather as gifts for tax purposes. In order to prove this claim, the respondent's financial expert utilized the Merton model to prove that, at the time of the advance, the petitioner, Dynamo, would have been expected to default on the loan. Applying the Merton model, the respondent's financial expert concluded that by Year 5, following the advance, the petitioner's probability of default was 43 percent. Therefore, a third-party lender would not make the advances in question. The expert's concluded default probabilities are summarized in Exhibit 3.

Ultimately, the court disagreed with the respondent's financial expert's conclusions.[9] This was in part influenced by the petitioner's financial expert's criticism of the respondent's application of the Merton model. Among numerous criticisms, the petitioner's expert primarily criticized the respondent's expert on three assumptions: asset volatility, return on assets and the time period over which the default probability was estimated. Correcting each assumption individually had varying impacts on the conclusions reached by the respondent, highlighting the sensitivity of the Merton model to small changes to its inputs. Moreover, collectively the methodological flaws had a significant impact on the conclusions reached for this analysis.

9 T.C. Memo. 2018-61. U.S. Tax Court. Docket Nos. 2685-11, 8393-12.

Beginning with asset volatility, the petitioner's expert criticized the respondent's analysis, principally on the respondent's use of a peer group's standard deviation of equity as a proxy for the petitioner's asset volatility. The respondent's first error was calculating the peer group's standard deviation of equity over long time periods ranging from four to 16 years. In addition, the petitioner argued that the asset mix of the subject company (Dynamo) was not comparable to the peer companies used by the respondent. For example, Dynamo's assets were weighted more toward cash rather than real estate. As previously mentioned, all else being equal, a company with assets that are primarily cash will be less volatile than a company whose assets are primarily real estate.

With that being said, assuming that the peer companies used by the respondent's expert to approximate Dynamo's asset volatility were appropriate, reducing the time period over which the volatility was calculated had a material impact on the estimated default probabilities. For example, changing only the time period by which the peer companies' equity volatility was calculated from 16 years to four years for all companies had the following impact on the respondent's results.

As shown in Exhibit 4, making this one adjustment had the impact of reducing Dynamo's cumulative probability of default in Year 5 from 43 percent to 26.2 percent. It is important to note that for Years 1 and 2 (when the Merton model is most relevant), making this one adjustment reduces Dynamo's probability of default basically to zero.

There were also methodological flaws in the respondent's return-on-assets calculation. As he did with his asset-volatility assumption, the respondent's financial expert relied on the peer group as a proxy for Dynamo's assumed return on assets. Once again, there are flaws with this approach due to the differing asset mix of Dynamo and the peer group. Moreover, the respondent's financial expert had fundamental flaws in his determination of the return on assets, including the varying time period by which he calculated the peer's return on assets and in the calculation itself. Only changing the time period by which the peer group's return on assets was calculated, even prior to fixing the fundamental flaw in the respondent's analysis, further reduced Dynamo's probability of default in Year 5 to 14 percent.

Finally, perhaps the most significant mistake made by the respondent in his analysis was the time period over which he calculated the petitioner's probability of default. As previously mentioned, the appropriate time period to apply the Merton model is no more than one year, because a lender would not go longer than one year without reviewing a borrower's audited financial statements and recalculate the probability of default. Furthermore, a built-in characteristic of the Merton model is that as time moves forward, the amplitude of the subject company's asset value increases dramatically, as the volatility variable is multiplied by time T (in the equation). Analyzing any company with this model, including the highest credit-quality firms, over long periods of time would render all companies likely to default on their debt obligations. Therefore, it is important to only apply the Merton model over a short period of time. In this case study, even if one accepted the respondent's analysis as presented, but only considered the cumulative probability of default in Years 1 and 2, one's conclusions would have drastically changed. For example, as shown in Exhibit 4, if one only applied the Merton model over two years, the estimated probability of default for Dynamo would have been 1.9 percent rather than 43 percent — a significant difference.

Ultimately, after considering the numerous methodological flaws in the respondent's financial expert's report, the judge concluded that,"[a]fter corrections for errors, [the respondent's financial expert's] report largely supported petitioner's position that Dynamo would have been able to borrow from a third-party lender."[10]

Conclusion

The Merton model is a mathematically accepted model used in academic research and by credit-rating agencies. When this methodology is applied properly, it provides a reasonable indication of a company's probability of default. However, it is our experience that in the context of bankruptcy and tax litigation, the model is often abused. Due to the sensitivity of the inputs outlined herein, a bankruptcy professional must proceed carefully when relying on or opposing the model in certain circumstances. **abi**

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10 *Id.*

Exhibit 3: The Expert's Default Probabilities

	Cumulative Probability of Default				
Scenario	Year 1	Year 2	Year 3	Year 4	Year 5
Respondent's Financial Expert	0.1%	1.9%	5.5%	9.5%	43.0%

Exhibit 4: The Expert's Default Probabilities: Asset Volatility Adjustment

	Cumulative Probability of Default				
Scenario	Year 1	Year 2	Year 3	Year 4	Year 5
Respondent's Financial Expert	0.1%	1.9%	5.5%	9.5%	43.0%
Petitioner - Adjusted Asset Volatility	0.0%	0.0%	0.4%	1.0%	26.2%